3.2 Compound & Double Angle Formulae

Question Paper

Course	CIE A Level Maths
Section	3. Trigonometry
Topic	3.2 Compound & Double Angle Formulae
Difficulty	Very Hard

Time allowed: 90

Score: /73

Percentage: /100

Question 1

(i) Prove that $\sin(A - B) = \sin A + \sin B$ is **not** true in general.

(ii) Find values for A and B, with $A \neq 0$ and $B \neq 0$, for which $\sin(A - B) = \sin A + \sin B$ **is** true.

[3 marks]

Question 2a

(a) Use the identities $\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$ and $\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$ to show that

$$\sin(X + Y - Z) \equiv$$
$$\sin X \cos Y \cos Z + \cos X \sin Y \cos Z - \cos X \cos Y \sin Z + \sin X \sin Y \sin Z$$

[3 marks]

Question 2b

(b) Hence show that $\sin(165°) = \frac{\sqrt{6}-\sqrt{2}}{4}$.

[4 marks]

Question 3

Show that

$$\tan 2A \equiv \frac{2\tan A}{1-\tan^2 A}$$

State clearly any trigonometric identities you use to show this result.

[4 marks]

Question 4

Given that $a \sin\theta + b \cos\theta$, where a and b are positive constants, is to be written in the form $R \sin(\theta + \alpha)$, find expressions for:

(i) α in terms of a and b

(ii) R in terms of a and b

[6 marks]

Question 5a

(a) Solve the equation

$$\cos 2\theta = \cos\theta \qquad 0 \leq \theta < 2\pi$$

[5 marks]

Question 5b

(b) Solve the equation

$$\tan 2x = 3 \tan x \qquad -\pi \leq x \leq \pi$$

[6 marks]

Question 6

Show that

$$\tan 2\theta \tan \theta \equiv \sec 2\theta - 1$$

[5 marks]

Question 7a

(a) Show that $5\sin\theta - 3\cos\theta$ can be written in the form $R\sin(\theta - \alpha)$ where $R = \sqrt{34}$, and $\alpha = 0.540$ radians correct to three significant figures.

[4 marks]

Question 7b

(b) Use your result from part (a), and the properties of the sine and cosine functions, to solve the equation

$$3\cos 2x + 5\sin 2x = 0.4 \qquad 0 \leq x \leq 2\pi$$

[5 marks]

Question 8a

(a) Use an identity for $\cos 2A$ to derive an identity for $\cos 4A$, in terms of $\cos A$.

[4 marks]

Question 8b

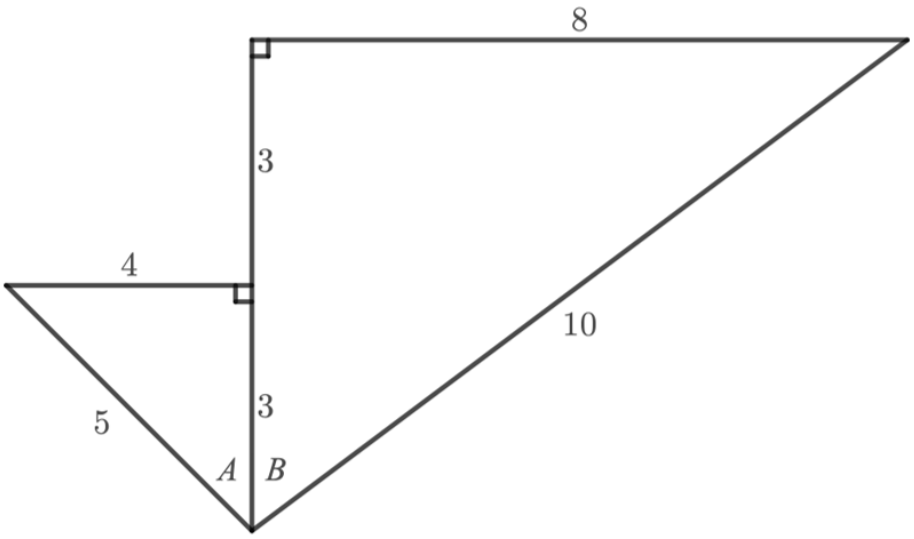
(b) Hence, or otherwise, solve the equation

$$2\cos 4x = 7\sin^2 x - 2 \qquad 0 \leq x \leq \pi$$

[5 marks]

Question 9

The diagram below shows two right-angled triangles.
Angles A and B have been labelled.



Given that $\alpha = A + B$, find the exact values of $\sin\alpha$, $\cos\alpha$ and $\tan\alpha$.

[7 marks]

Question 10

(i) Explain briefly why $\theta = 0$ is **not** a solution to the equation $3\theta \cot 2\theta = 0$.

(ii) By using an appropriate approximation, determine the value of

$$\lim_{\theta \to 0} 3\theta \cot 2\theta$$

[4 marks]

Question 11a

The alternating voltage, V, in a domestic electrical circuit, t seconds after it is switched on is modelled by the function

$$V = 115 \sin \omega t + 115\sqrt{3} \cos \omega t.$$

(a) Express

$$115 \sin \omega t + 115\sqrt{3} \cos \omega t$$

in the form

$$R \sin(\omega t + \alpha)$$

where R and α are constants to be found. $R > 0$ and is α acute.

[2 marks]

Question 11b

In the UK, domestic electricity runs at a frequency, f, of 50 Hertz (Hz).
The constant ω, is given by $\omega = 2\pi f$.

(b) (i) Find the initial voltage when a domestic appliance (such as a kettle or TV) is switched on.

(ii) Find the time at which the voltage first turns negative.

[4 marks]

Question 11c

(c) (i) Find the period of one cycle of voltage in the UK.

(ii) In the US, the period of one cycle is $\frac{1}{60}$ seconds.

Write down the frequency of US domestic electricity.

[2 marks]